Hall Effect & Magnetoresistance

Edwin Hall 1879 – a current-carrying wire experiences a force when an external magnetic field is applied ($\perp$ to $\vec{I}$)

- is the force imparted on the entire wire, or just the charge carriers?

Note: At this time, the $\bar{e}$ had not yet been discovered!

Hypothesis: the force is imparted only on the carriers. They should be drawn to one side of the wire, increasing their path length through the material ∴ increase resistance

magnetoresistance

Recall: A charge in a magnetic field

S.I. units:

$$\bar{B} = \mu_0 \bar{H} + \bar{M}$$

cgs units: (old & dumb)

$$\bar{B} = \bar{H} + 4\pi \bar{M}$$

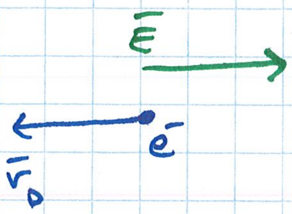
A & M is old (but not dumb) ∴ uses cgs

I will (try to) use S.I.

In S.I. units: $\bar{E}$ & $\bar{B}$ can be defined such that the Lorentz Force law is correct:

$$\boxed{\bar{F} = q(\bar{E} + \bar{v} \times \bar{B})}$$

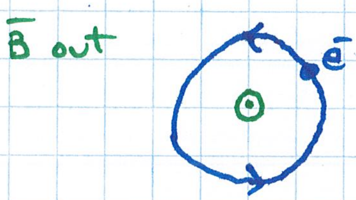
→ just $\bar{E}$ field:



$$\bar{F} = -e\bar{E}$$

$$\bar{j} = -en\bar{v}_D$$

→ just $\bar{B}$ field:



centripetal force

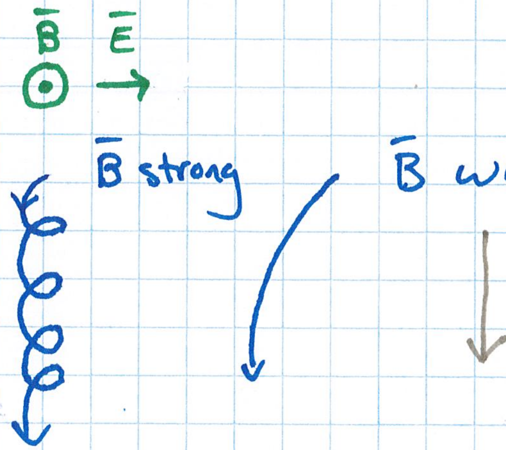
$$\frac{mv^2}{r} = evB$$

Lorentz force

$$\omega_c = \frac{v}{r} = \frac{eB}{m}$$

cyclotron frequency

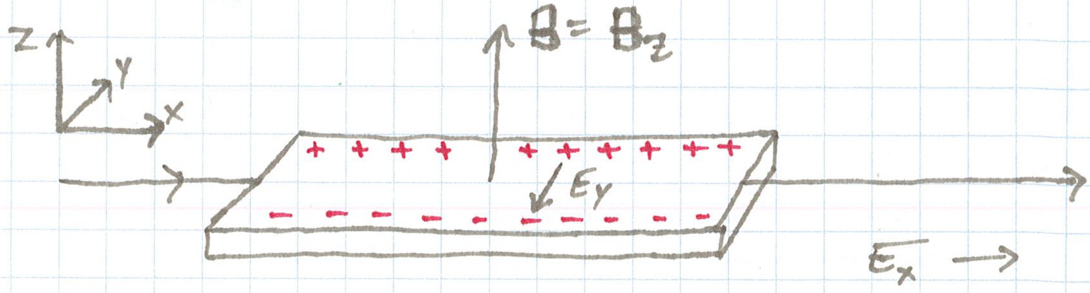
→ both $\bar{E}$ & $\bar{B}$:



$$\bar{F} = -e(\bar{E} + \bar{v}\bar{B})$$

- drift $\perp$ to $\bar{E}$ & $\bar{B}$

Hall Effect



- in s.s. E_y balances Lorentz force
- two quantities of interest:

magnetoresistance: $\rho(B) = \dfrac{E_x}{j_x} = \rho_{xx}$

Hall coefficient: $R_H = \dfrac{E_y}{j_x B_z}$ $\qquad \rho_{yx} = \dfrac{E_y}{j_x}$

Drude eqn of motion:

$$\frac{d\bar{p}}{dt} = -e\left(\bar{E} + \frac{\bar{p}}{m} \times \bar{B}\right) - \frac{\bar{p}}{\tau}$$

steady state: $\dfrac{d\bar{p}}{dt} = 0$

$$\hat{x} \rightarrow 0 = -eE_x - \frac{e p_y B}{m} - \frac{p_x}{\tau}$$

$$\hat{y} \rightarrow 0 = -eE_y + \frac{e p_x B}{m} - \frac{p_y}{\tau}$$

$$\hat{z} \rightarrow 0 = -E_z - \frac{p_z}{\tau}$$

Recall:

$\omega_c = \dfrac{eB}{m}$

$\sigma_0 = \dfrac{ne^2\tau}{m}$ ↳ DC cond.

$\therefore$ magnetoresistance:

$$\rho(B) = \rho_{xx} = \frac{E_x}{j_x} = \frac{1}{\sigma_0}$$

$$\rho_{yx} = \frac{E_y}{j_x} = -\frac{\omega_c \tau}{\sigma_0}$$

Hall coefficient:

$$R_H = \frac{\rho_{yx}}{B} = \frac{E_y}{B\, j_x}$$

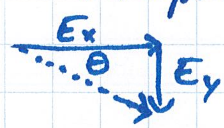
$$\omega_c = \frac{eB}{m}$$

$$\sigma_0 = \frac{ne^2\tau}{m}$$

$$R_H = \frac{-\omega_c \tau}{B\, \sigma_0}$$

$$\boxed{R_H = -\frac{1}{ne}}$$

Interpretation of ω_c



$$\tan\theta = \frac{E_y}{E_x} = -\omega_c \tau$$

θ_H is a measure of rotation before scattering

Important: $\rightarrow$ R_H is independent of τ

$\rightarrow$ R_H depends on n (carrier density)

$\rightarrow$ R_H sensitive to charge of carriers (electrons or holes....)

We can measure R_H!!

↳ need E_y, j_x, & B

multiply by $-\frac{ne\tau}{m}$ & rearrange for E_i:

Note: $j_i = -\frac{ne}{m} p_i$

$$\sigma_0 E_x = \omega_c \tau j_y + j_x$$
$$\sigma_0 E_y = -\omega_c \tau j_x + j_y$$
$$\sigma_0 E_z = j_z$$

$$\begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix} = \underbrace{\frac{1}{\sigma_0}\begin{pmatrix} 1 & \omega_c\tau & 0 \\ -\omega_c\tau & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}}_{\rho \text{ is now a tensor}} \begin{pmatrix} j_x \\ j_y \\ j_z \end{pmatrix}$$

- For Hall effect → enforce $j_y = 0$ in s.s.

$$\therefore \quad E_x = \frac{1}{\sigma_0} j_x$$

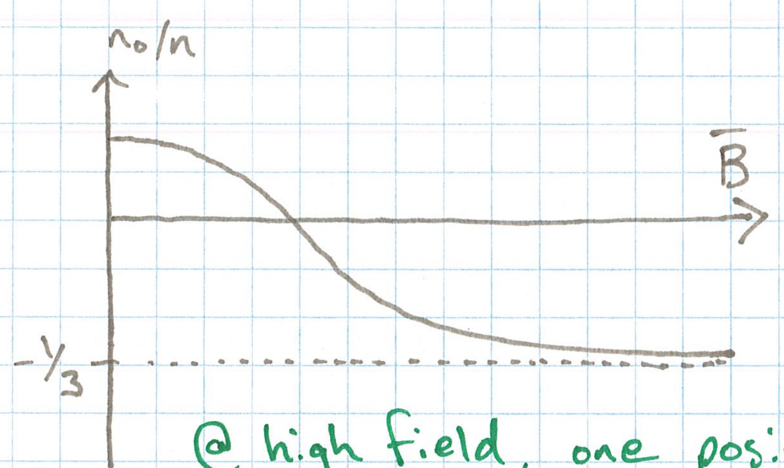
$$E_y = -\frac{\omega_c \tau}{\sigma_0} j_x$$

By measuring R_H we can deduce the carrier density (n) & the sign of charge of the carriers (should be (−) right...?)

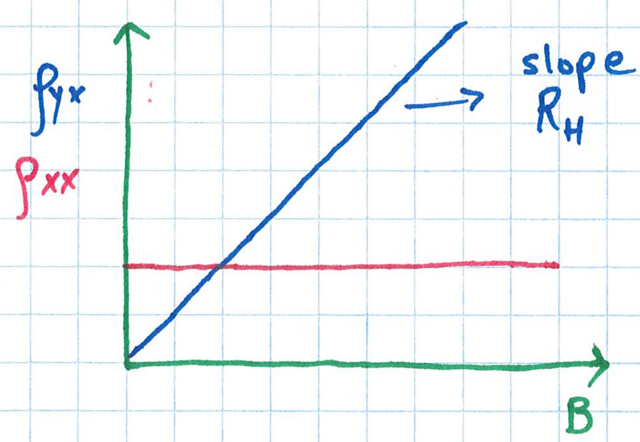
→ A & M Table 1.4

Fig 1.4

Aluminum



@ high field, one positive carrier per atom



real life

Drude predicts: → no field dependence, B ✗
for R_H → no temp. dependence, T ✗
→ no τ dependence made up anyway!

→ also depends on preparation (purity)